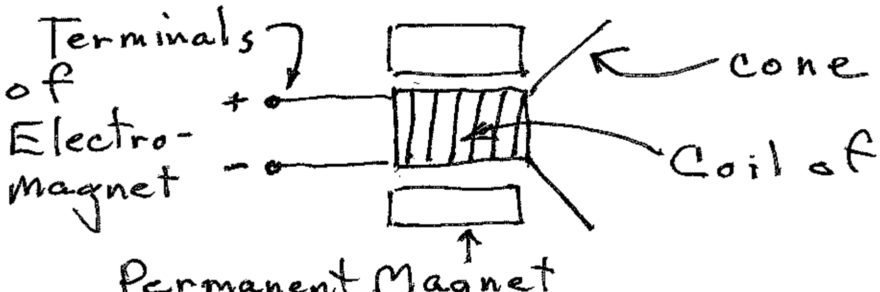
The Concept of Speaker Impedance and the Associated Analysis.

Kenny Lanne

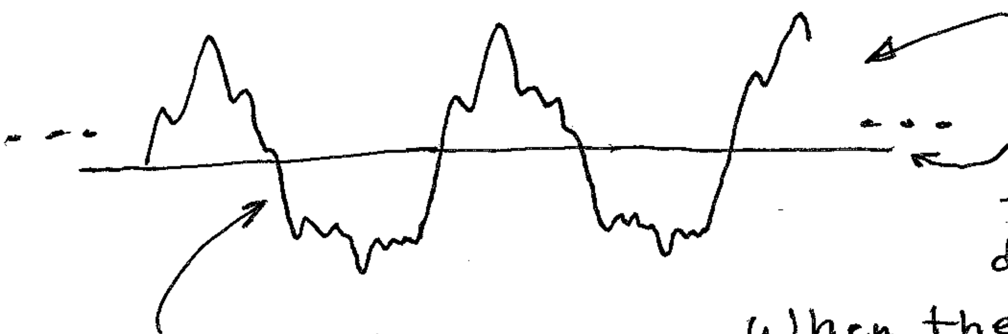
NOTE: A knowledge of basic circuit theory and complex numbers is required.

Not long after one begins the study of Physics or Engineering, the concepts of work & power are introduced. It has been a long-standing fact of Physics that no real work can be done with reactive loads. Impedances are composed of Real & Reactive parts. And so it is with Loudspeakers. The loudspeaker consists of an electromagnet (a coil with current through it) placed in, or submerged in, the magnetic field of a permanent magnet.



Speaker Cross-section

An audio signal, by definition, contains AC (alternating current). A signal such as my voice will create currents in an amplifier that will travel "back & forth" or "alternate" in exact accordance with the characteristics of my voice.



When the signal is above the line, current flows in one direction →.

When the signal is below the line, current flows in the opposite direction ←.

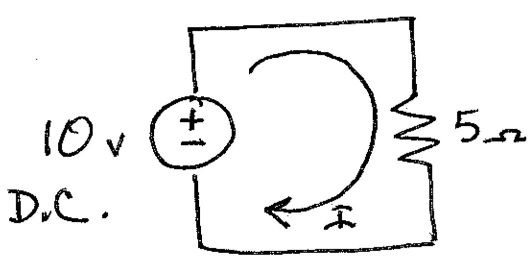
Hypothetical Voice Signal

At very low (slow) frequencies, we can actually see the speaker moving back & forth.

The amount of electrical power transfered to the speaker is related to the Impedance (Z) of the speaker. However, when we are told a speaker has an impedance of 4, 8, or 16 ohms, this is only a nominal value. It is NOT constant across the

the frequency band. It stands to reason that if the impedance is not constant across all frequencies, then power delivery is not constant across all frequencies. Page 4 shows the impedance of an Eminence speaker (dashed line). Let's look at Power.

A very simple power calculation for a voltage source and a resistor connected to it are shown below.



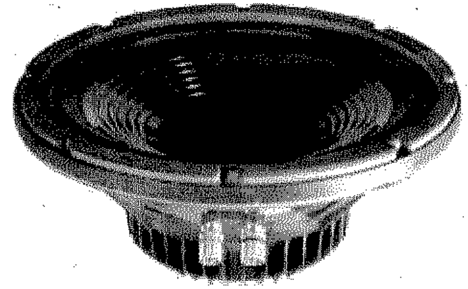
$I = \text{current} = \frac{10v}{5\Omega} = 2\text{amps}$

$\frac{V}{R} = I$ ⇐ Ohm's law

$\text{Power} = V \cdot I = 10v \cdot 2A = 20\text{watt}$

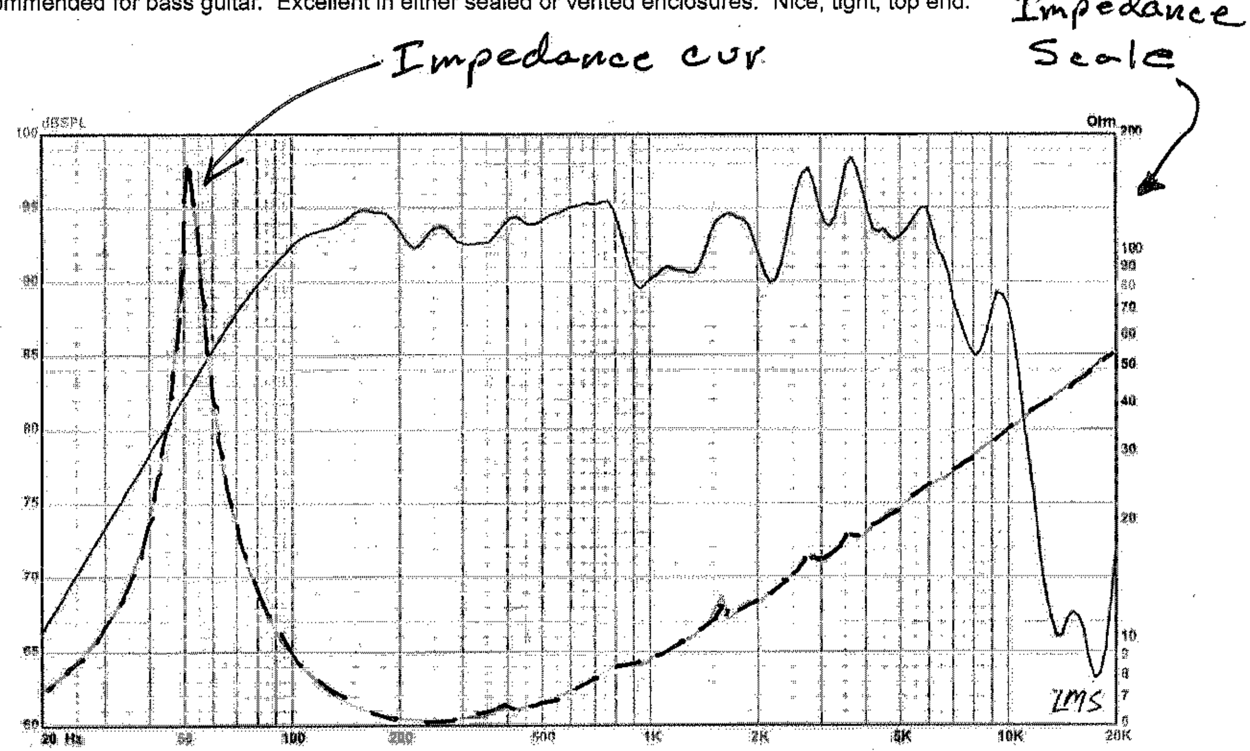
But what about an A.C. signal? The actual power vs. frequency can be much more difficult to calculate. However, we can calculate the power for an A.C. sinewave. A sinewave is the purest tone we can have. This is a natural law of physics & therefore we know there are NO other frequencies present except for our sinewave frequency.

EMINENCE®
The Art and Science of Sound



BASSLITE® CA2010

Recommended for bass guitar. Excellent in either sealed or vented enclosures. Nice, tight, top end.



Specification

Nominal Basket Diameter	10", 254mm
Nominal Impedance*	8 ohms
Power Rating**	150W
Resonance	51Hz
Usable Frequency Range	48Hz-7kHz
Sensitivity***	93.5
Magnet Weight	4 oz.
Gap Height	0.28", 7.2mm
Voice Coil Diameter	2", 50.8mm

Thiele & Small Parameters

Resonant Frequency (fs)	51Hz
DC Resistance (Re)	5.46
Coil Inductance (Le)	0.55mH
Mechanical Q (Qms)	17.5
Electromagnetic Q (Qes)	0.52
Total Q (Qts)	0.51
Compliance Equivalent Volume (Vas)	49 liters / 1.7 cu. ft.
Peak Diaphragm Displacement Volume (Vd)	126cc
Mechanical Compliance of Suspension (Cms)	0.27mm/N
BL Product (BL)	11.2 T-M 11.2
Diaphragm Mass inc. Airload (Mms)	37 grams 37g
Efficiency Bandwidth Product (EBP)	98
Maximum Linear Excursion (Xmax)	3.5mm
Surface Area of Cone (Sd)	360.7 cm2
Maximum Mechanical Limit (Xlim)	7.0mm

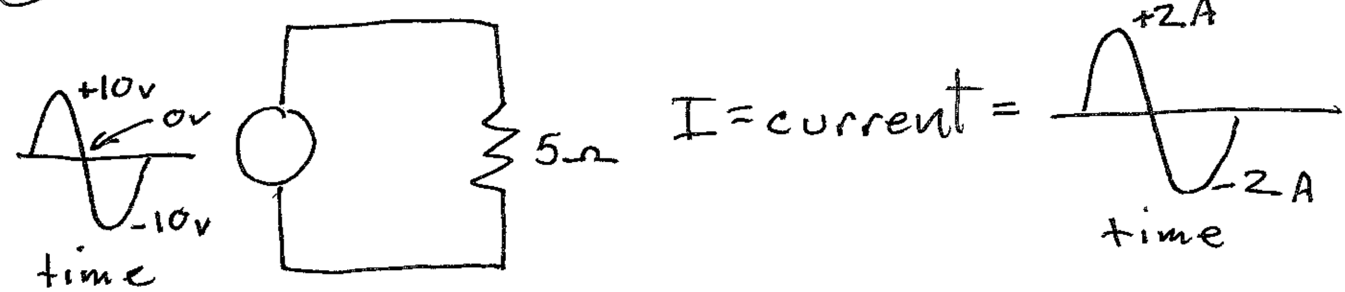
Mounting Information

Recommended Enclosure Volume	
Sealed	17-23 liters/ 0.6-0.8 cu.ft.
Vented	23-54 liters/ 0.8-1.9 cu.ft.
Overall Diameter	10.25", 260.4mm
Baffle Hole Diameter	9.13", 231.9mm
Front Sealing Gasket	fitted as standard
Rear Sealing Gasket	fitted as standard
Mounting Holes Diameter	0.28", 7.0mm
Mounting Holes B.C.D.	9.73", 247.1mm
Depth	4.75", 121mm
Net Weight	3.6 lbs.,1.6 kg
Shipping Weight	4.7 lbs., 2.1 kg

Materials of Construction

- Copper voice coil
- Polyimide former
- Neodymium magnet
- Non-vented core
- Die-cast aluminum basket
- Aluminum Alloy Cone
- Cloth cone edge
- Solid composition felt dust cap

* Please inquire about alternative impedances.
** Multiple units exceed published rating evaluated under EIA 426A noise source and test standard while in a free-air, non-temperature controlled environment.
*** The average output across the usable frequency range when applying 1W/1M into the nominal impedance. Ie: 2.83V/8ohms, 4V/16ohms.
Eminence response curves are measured under the following conditions: All speakers are tested at 1w/1m using a variety of test set-ups for the appropriate impedance | LMS using 0.25" supplied microphone (software calibrated) mounted 1m from wall/baffle | 2ft. X 2ft. baffle is built into the wall with the speaker mounted flush against a steel ring for minimum diffraction | Hafler P1500 Trans-Nova amplifier | 2700 cu.ft. chamber with fiberglass on all six surfaces (three with custom-made wedges)



The introduction of an A.C. signal has slightly complicated our analysis. First of all, we no longer have 10v across the resistor at all times. In fact, for some instances we have 0v across the resistor. Now -10 volts creates just as much power as +10 volts, so the change in polarity is not the issue. The issue is that the voltage and current are constantly changing at the rate of our sinewave. Because of this, we no longer have 20 watts across our resistor. We need to take the RMS (root-mean square) values of our voltage and current. Without going through the necessary calculus, let's just agree that the RMS value of a sinusoidal current or voltage is 0.707 times the peak. This is <u>ONLY</u> true for sinewaves and not more complicated waveforms!!!

Let's perform the power calculation with RMS values.

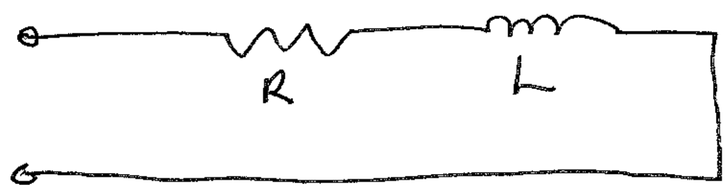
⑥ [.707 x 10 volts x .707 x 2 amps] = 10 watts.

This is only ½ the power calculated for the D.C. case. Furthermore, this is for only one frequency with a constant resistor. As we discussed, and as shown on page 4, the impedance of a speaker is NOT a constant resistance. You may guess that the actual power delivered to the speaker at each frequency from 20 Hz to 20 KHz can be a much more complicated problem, and will not be solved in these notes. But how is it the impedance of a speaker varies so much vs. frequency

---

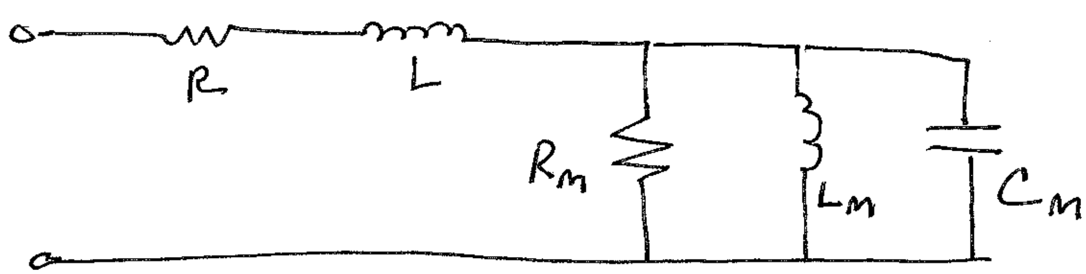
The impedance varies so much because the speaker is a "domain transformer" so to speak. It transforms electrical power to the mechanical domain and then to the acoustical domain. Likewise, the impedance we see across the electrical terminals is not just due to the wire that makes up the voicecoil. The mechanical and acoustical circuit

(7) elements get transformed back into the electrical domain, and the amplifier must contend with these other circuit elements, although sometimes the acoustical elements can be ignored because speakers can be very inefficient devices. Instead of the electrical model containing simply an inductor (coil) and some wire resistance,



R = resistor
L = inductor

It looks more like the following



R_m = mechanical losses

L_M = Inductance due to the mechanical spring constant of the speaker.

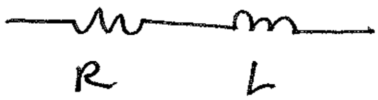
C_m = Capacitance due to the mechanical mass of the cone, voice coil, & air load.

⑧ So what values do the mechanical elements have? To solve this, we must be familiar with transformation concepts and for the sake of brevity we will define them & accept them. The transformations come from the Thiele-Small parameters developed by Richard Small and Neville Thiele. For more on this, visit the AES (Audio Engineering Society www.aes.org) They have most of Small's & Thiele's original papers & research.

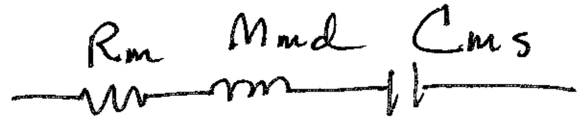
On page 4, we have a Thiele-Small paremeter called BL. This is the product of the magnetic flux-density (B) and the length of the wire in the voice coil embedded in the magnetic flux. The transformation of mechanical impedance to electrical impedance is actually quite simple using the BL product. It's also important to note their is a sign change when the impedance transformation occurs. We don't have time to cover all of the circuit theory laws but suffice it to say the original mechanical circuit & the electrical transformation of that circuit are what we call circuit DUALS.

⑨ Circuit DUALS have the signs of all their reactances changed and all series branches of the circuit are converted to parallel branches and vice versa.

First, the electrical & mechanical models



electrical



mechanical
speaker model

To combine the two we most transform the electrical to mechanical or the mechanical to electrical. We will do the latter. The mechanical values are given as thiele-Small parameters:

$C_{ms} = .00027 \frac{m}{N}$ $\frac{\text{(meters)}}{\text{(Newton)}}$

$M_{md} = .037$ kg

Rm is found from the mechanical Q at the resonant frequency, f_s, which is also given in the Thiele-Small parameters as 51 Hz.

For series resonance

$Q_M = 17.5 = \frac{2\pi f M_{md}}{R}$ OR $\frac{2\pi(51)(.037)}{R}$; $R = 0.68 \frac{kg}{sec}$

(10) The transformation from mechanical Z to electrical is given as:

$$Z_e = \frac{(BL)^2}{Z_m}$$

$BL = 11.2$ from page 4

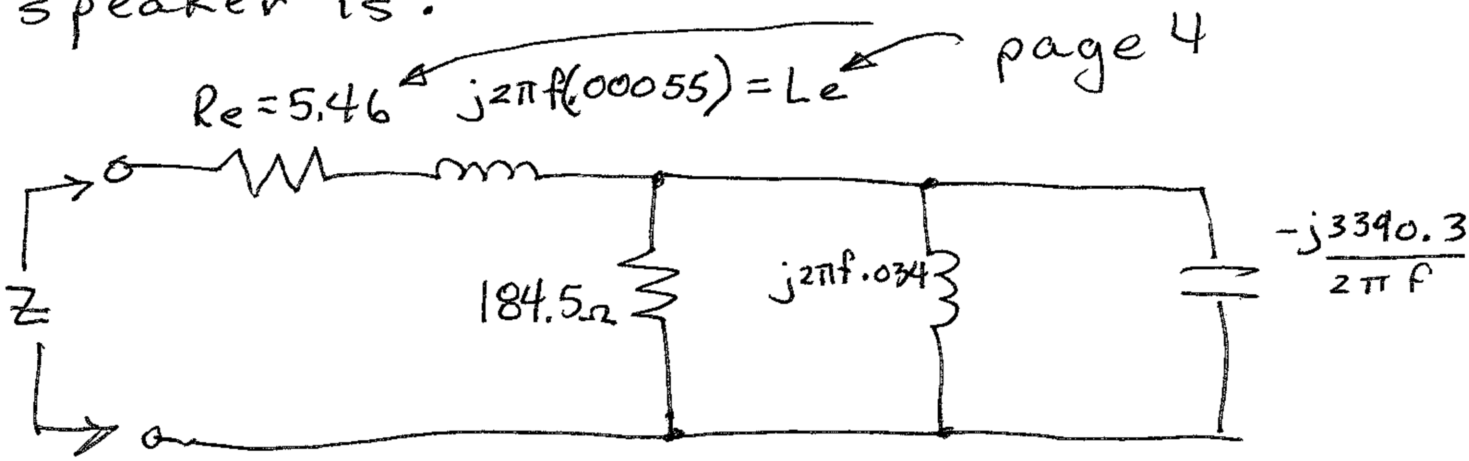
Therefore

$$R_m = \frac{(11.2)^2}{0.68} = 184.5\,\Omega$$

$$C_m = \frac{(11.2)^2}{j2\pi(freq)(.037)} = -j\,\frac{3390.3}{2\pi(freq)}$$

$$L_m = \frac{(11.2)^2}{\dfrac{1}{j2\pi(freq)(.00027)}} = j2\pi(freq).034$$

Now our complete and final model for the electrical impedance of a speaker is:



To solve for the total impedance we add Re & Le to the parallel combination of Rm, Lm, & Cm. Circuit theory dictates that the impedance of 3 parallel elements is the reciprocal of the sum of the reciprocals. Or more clearly stated:

$$Z_{TOT} = \frac{1}{\frac{1}{R_m} + \frac{1}{X_{Lm}} + \frac{1}{X_{Cm}}}$$

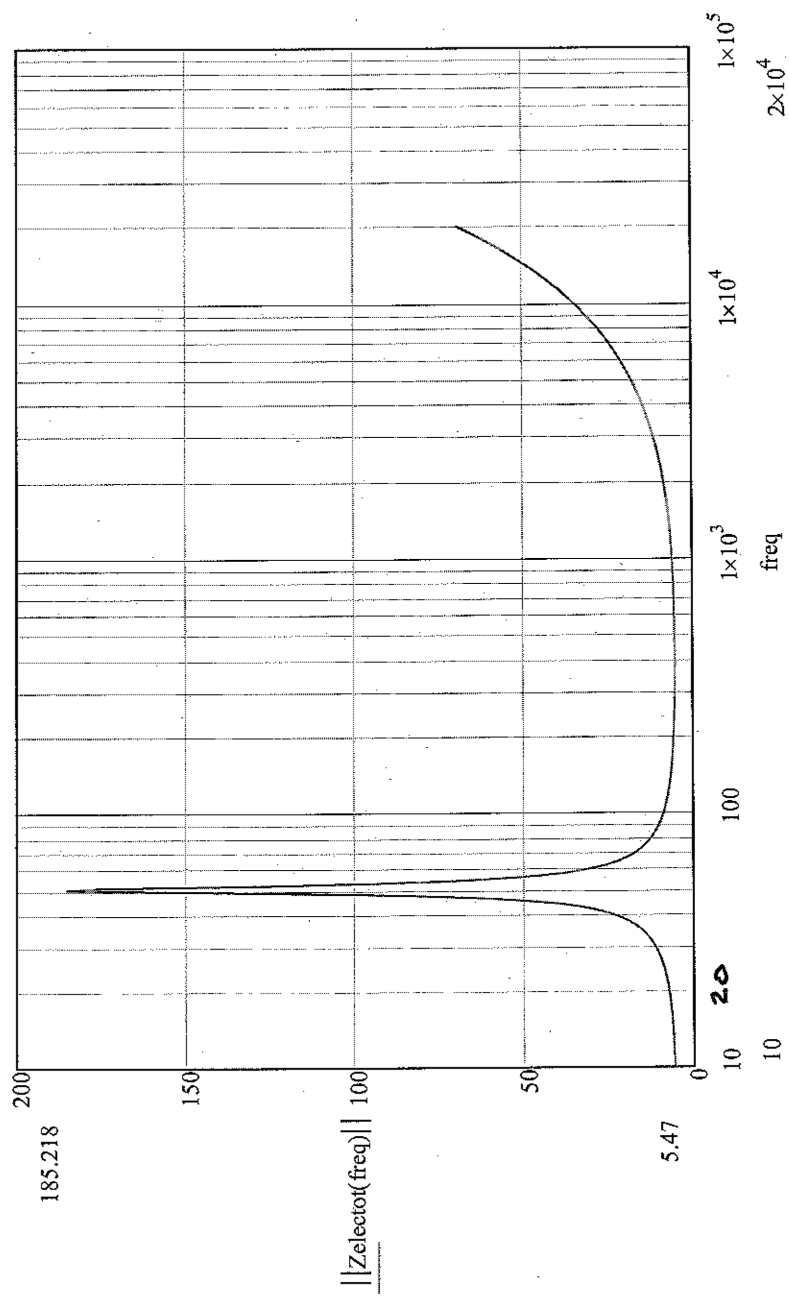
Therefore the total impedance is given by:

$$Z_{TOT} = Re + j2\pi freq\, Le + \frac{1}{\frac{1}{R_m} + \frac{1}{\frac{-j3390.3}{2\pi freq}} + \frac{1}{j2\pi freq(.034)}}$$

If we use a computer program to sweep the variable "freq" from 20Hz to 20KHz we get the magnitude plot shown on the next page. I used MathCad software to plot the data. Notice it matches the plot on page 4. We have a resonant peak at 51 Hz with a magnitude just below 200Ω (185Ω). At 20KHz we have a magnitude above 50Ω & less than 75Ω. Is it any wonder musicians can get such a variety of tones with different amp & speaker combination

(12)

There's a lot going when a speaker is interfaced with an amp.



MathCad Model of Speaker Impedance for Eminence CA2010.

References: ① Engineering Circuit Analysis, Hayt & Kemmerly McGraw Hill

② Direct-Radiator Loudspeaker System Analysis, Richard H. Small, IEEE Transactions on Audio and Electro-acoustics, vol AU-19, pp 269-281, Dec. 1971